The Geometry of the MRB constant by Marvin Ray Burns

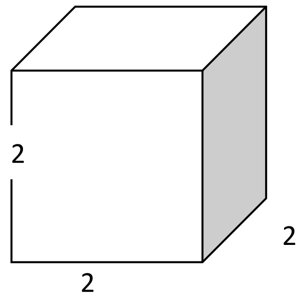
Abstract

The MRB constant is the upper limit point of the sequence of partial sums defined by S(x)=sum((-1)^n*n^(1/n),n=1..x). The goal of this paper is to show that the MRB constant is geometrically quantifiable. To "measure" the MRB constant, we will consider a set, sequence and alternating series of the nth roots of n. Then we will compare the length of the edges of a special set of hypercubes or n-cubes which have a content of n. (The two words hypercubes and n-cubes will be used synonymously.) We will finally look at the value of the MRB constant as a representation of that comparison, of the length of the edges of a special set of hypercubes, in units of dimension 1/ (units of dimension 2 times units of dimension 3 times units of dimension 4 times etc.). For an arbitrary example we will use units of length/ (time*mass* density*…).

A Countable Infinite Set

Consider, r, the set of roots of positive integers of the form r = n^(1/n). Of course the elements of this set are of the form x^(1/y). However what is not obvious is the geometric interpretation of x^(1/y). At least as far as natural number valued x and y>0 are concerned, x represents the content of an n-cube and y represents its dimension. For instance we take a cube of any given content and find the length of one of its sides. Let's suppose the volume was 8 units^3. What would be the length of one of its edges? We might easily deduce that the length is 2. To confirm this answer we simply construct a cube of 2 linear units in length as in Diagram 1 and find its volume.

Diagram 1



The volume in units^3 of the cube in Diagram 1 is indeed 2*2*2 = 8.

Now we look at the previous sentence with x^(1/y) in mind. 8^(1/3) = 2 implies the volume of the cube in Diagram 1 raised to the power of the reciprocal of its dimension equals the length of one of its sides. That is a geometric interpretation of x^(1/y) as far as natural number valued x and y>0 are concerned.

Open Question

What is the geometric interpretation of x^(1/y) for all real values of y>0?

Sequence and Alternating Series

Now we will consider the sequence of roots of positive integers of the form r = {n^(1/n)} = {1^(1/1), 2^(1/2), 3^(1/3), ...}. Then we will add the elements of r in the alternating series L=

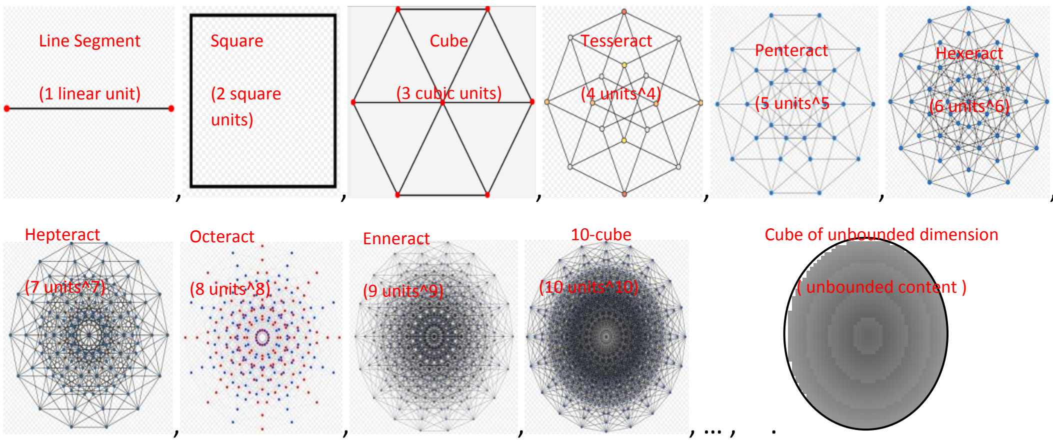
$$\sum_{n=1}^{\infty} (-1)^n r(n) = \sum_{n=1}^{\infty} (-1)^n n^{\frac{1}{n}}.$$

Concerning the partial sums of L, we remember S(x)= $\sum_{n=1}^{x} (-1)^n n^{\frac{1}{n}}$, and we find, S(x) is divergent as x goes to infinity. However S(2x) and S(2x+1) both converge as x goes to infinity and the difference between S(2x) and S(2x+1) also converges as x goes to infinity.

Special Hypercubes

In geometric terms: As in Diagram2, we give each n-cube a hyperbolic volume (content) equal to its dimension [1],

Diagram2



so that we have a line segment of 1 linear unit, a square of 2 square units, and so forth.

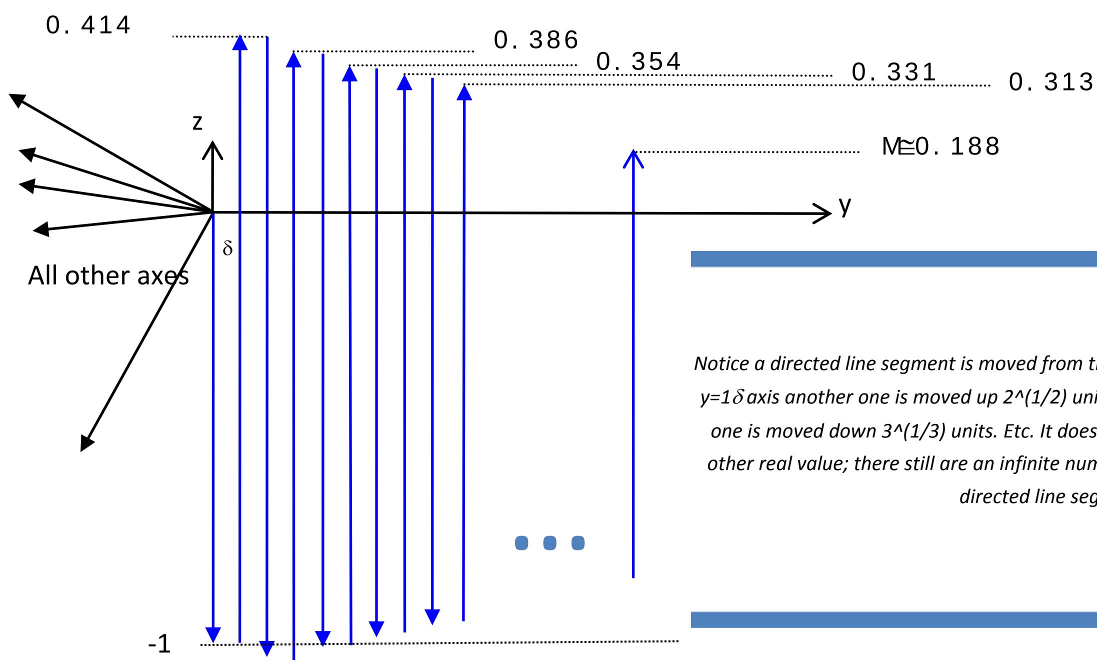
The last cube in the diagram may be just an invention of the imagination because what would be an example of a hypercube of unbounded dimension with unbounded content? Furthermore, there seems to be a paradox invoked when taking n-cubes as n->infinity. While n has a numeric value the content of the n-cube or hypercube is defined in the specific unit of choice, whether the unit is inches, meters or what not and the resulting length of an individual edge is also defined in the same unit and is computed as shown above. "The [content] of the [n-]cube raised to the power of the reciprocal of its dimension equals the length of one of its [edges]. " However, when we arrived at the hypercube of unbounded dimension, where n no longer has a numeric value, the assigned "unbounded content" could be meant to be in units of feet, let's say; while the resulting length of an individual edge is $\lim_{u\to\infty} u^{\frac{1}{u}}$ =1 which could be in feet or any other unit of length because there are the same amount of inches or meters or any other unit of length in infinity feet as there are feet. To avoid the resulting ambiguity we will look at the sequence of roots of positive integers of the form r = {n^(1/n)} = {1^(1/1), 2^(1/2), 3^(1/3), ...} as being analogous to the clopen interval [1, infinity).

A Sum of the Series

Above it is mentioned, “Add the elements of r in the alternating series L= $\sum_{n=1}^{\infty}(-1)^n r(n)$

= $\sum_{n=1}^{\infty}(-1)^n n^{\frac{1}{n}}$.“ To show that geometrically we do the following: as in Diagram3, on the y,z-plane line up an edge of each n-cube or hypercube. The numeric values displayed in the diagram are the partial sums of S(2*x) where S(x)=sum((-1)^n*n^(1/n),n=1..x).

Diagram3



Notice a directed line segment is moved from the origin down the z-axis. Then at the y=1δ axis another one is moved up 2^(1/2) units. Then at the y=2δ axis yet another one is moved down 3^(1/3) units. Etc. It does not matter whether δ is one or any other real value; there still are an infinite number of y-valued axes with matching directed line segments

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We may say metaphorically that Diagram3 is the path along the units of a particle moved 1 inch down in 2^(1/2) seconds, losing 3^(1/3) units of mass with density that increases 4^(1/4) units etc. The resulting position of the particle is represented by M = $\lim_{u\to\infty}\left(\sum_{n=1}^{2u}(-1)^n n^{\frac{1}{n}}\right)$.

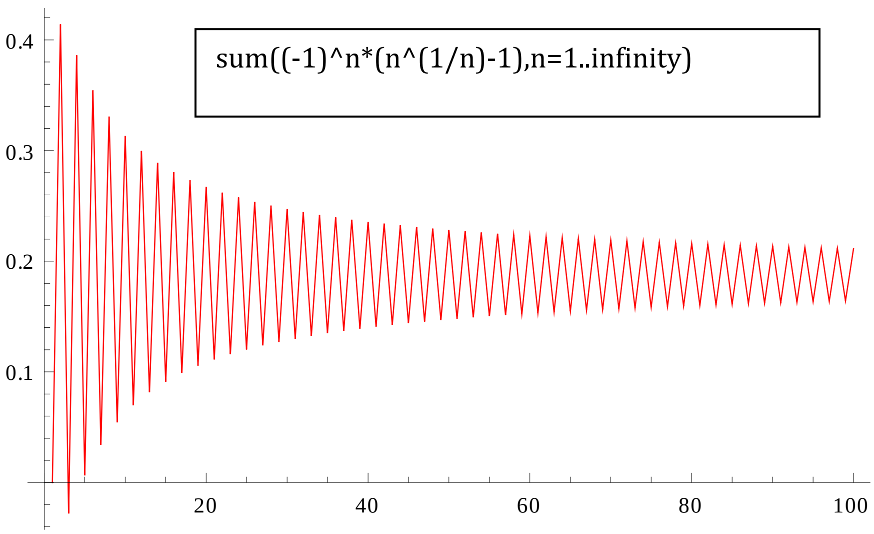
The MRB Constant

As the dimension and the content of a hypercube, both go to infinity we have the following: First in Diagram 2 the difference between the length of an edge of the hypercube with content 2n, and an edge of the hypercube with content 2n+1, goes to the constant value $\lim_{n\to\infty}\left((2n+1)^{\frac{1}{2n+1}}-(2n)^{\frac{1}{2n}}\right)$ = 0; so as n goes to infinity, the length of an edge of the hypercube with content 2n and an edge of the hypercube with content 2n+1 become closer to being the same. Second in Diagram 3 an edge of each n-cube is arranged on y-valued axes in such a way that M= $\sum_{n=1}^{\infty}\left((2n+1)^{\frac{1}{2n+1}}-(2n)^{\frac{1}{2n}}\right)$ = $\lim_{u\to\infty}\left(\sum_{n=1}^{2u}(-1)^n n^{\frac{1}{n}}\right)$. M is the MRB constant[2].

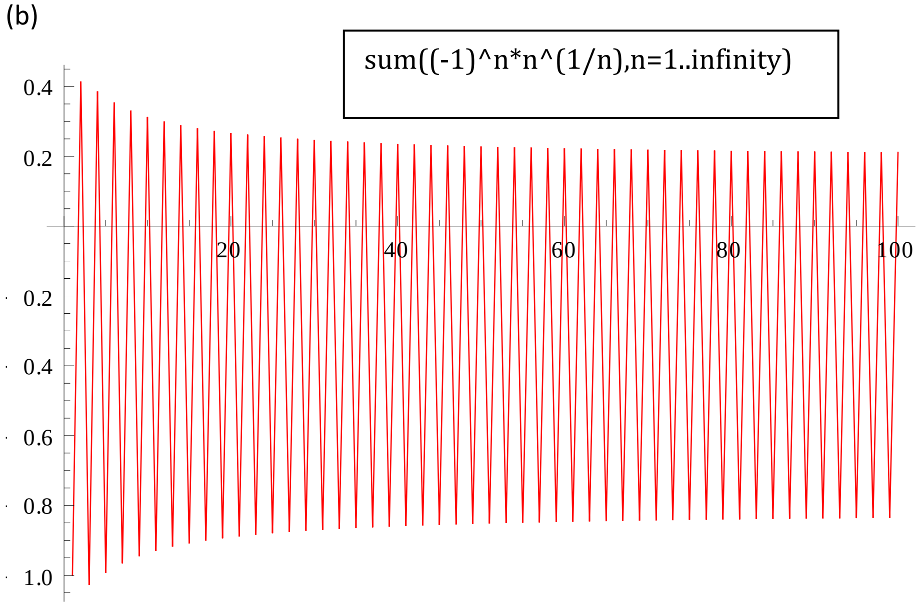
A numerical approximation of M can be computed by the following summation sum((-1)^n*(n^(1/n)-1),n=1..infinity),[3] which sum converges (See diagram 4.), while sum((-1)^n*n^(1/n),n=1..infinity) [4] diverges, as mentioned above. One should use acceleration methods when computing a numerical approximation of the MRB constant because it can be shown that one must sum a number in the order of 10^(n+1) iterations of (-1)^n*(n^(1/n)-1) to get n accurate digits of the MRB Constant. However, using a convergence acceleration of alternating series algorithm of Cohen-Villegas-Zagier one can compute the first 60 digits in only 100 iterations[5].

Diagram 4

(a)



(b)



In diagram 4a both the lim sup[6] and the lim inf converge upon the MRB constant, while in 4b only the lim sup converges upon it with the lim inf converging upon MRB constant-1. The MRB constant is Sloane's On-Line Encyclopedia of Integer Sequences id:A037077. More information, including a brief but documented history, can be found in Wikipedia[7].

Summary

In retrospect, the geometry used here, particularly in diagram3, is transdimensional and thus we find it hard to understand through the previous experiences of our senses. (To examine its geometry we used edges from hypercubes of all dimensions.) However, considering the various temporal-spatial dimensions proposed in some theories,[8] is there some significance to the MRB constant in our daily lives? Nevertheless, we have seen that the value of the MRB constant is geometrically quantifiable; it is the lim sup of the sequence that represents a particle traveling along a directed line segment that is moved 1 unit from the origin down the z-axis. Then at the y=1 axis another one is moved up 2^(1/2) units. Then at the y=2 axis yet another one is moved down 3^(1/3) units. Etc. The resulting z value of the particle's position M = $\lim_{u \to \infty} \left(\sum_{n=1}^{2u} (-1)^n n^{\frac{1}{n}} \right)$.

References

[1]

http://www23.wolframalpha.com/input/?i=n-cube

[2]

S. R. Finch, Mathematical Constants, Cambridge, 2003, p. 450.

[3]

http://mathworld.wolfram.com/MRBConstant.html

[4]

http://mathworld.wolfram.com/notebooks/Constants/MRBConstant.nb

[5]

http://arxiv.org/abs/0912.3844

[6]

http://en.wikipedia.org/wiki/Upper_limit

[7]

http://en.wikipedia.org/wiki/MRB_constant

[8]

http://arxiv.org/abs/hep-ph/9803466